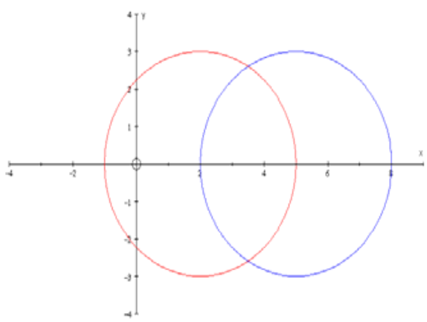
June 2005
6673 Pure P3
Mark Scheme

Question Number	Scheme	Marks
1. (a)	Finding f (± 2), and obtaining $16 - 32 + 10 + 6 = 0$	M1, A1
	Or uses division and obtains $2x^2 - kx...$,	M1
	obtaining $2x^2 - 4x - 3$ and concluding remainder = 0	A1
(b)	Finding f ($\pm\frac{1}{2}$), and obtaining $-\frac{1}{4} - 2 - \frac{5}{2} + 6 = 1\frac{1}{4}$	M1, A1
	Or uses division and obtains $x^2 - kx...$,	
	obtaining $x^2 - \frac{9}{2}x + \frac{19}{4}$ and concluding remainder = $\frac{5}{4}$	
(c)	$x = 2$ (also allow $\frac{2 \pm \sqrt{10}}{2}$ or $\frac{4 \pm \sqrt{40}}{4}$)	B1 (5)
2. (a)	Writes down binomial expansion up to and including term in x^3, allow ${}^nC_r$ notation	
	$1 + nax + n(n-1)\frac{a^2x^2}{2} + \frac{n(n-1)(n-2)}{6}a^3x^3$ (condone errors in powers of a)	M1
	States $na = 15$	B1
	Puts $\frac{n(n-1)a^2}{2} = \frac{n(n-1)(n-2)a^3}{6}$ (condone errors in powers of a)	dM1
	$3 = (n - 2)a$	
	Solves simultaneous equations in n and a to obtain $a = 6$, and $n = 2.5$	M1 A1 A1 (6)
	[n.b. Just writes $a = 6$, and $n = 2.5$ following no working or following errors allow the last M1 A1 A1]	
(b)	Coefficient of $x^3 = 2.5 \times 1.5 \times 0.5 \times 6^3 \div 6 = 67.5$	B1 (1)
	(or equals coefficient of $x^2 = 2.5 \times 1.5 \times 6^2 \div 2 = 67.5$)	[7]

Question Number	Scheme	Marks
3. (a)	Attempt at integration by parts, i.e. $kx\sin 2x \pm \int k\sin 2x\,dx$, with $k = 2$ or ½	M1
	$= \frac{1}{2}x\sin 2x - \int \frac{1}{2}\sin 2x\,dx$	A1
	Integrates sin 2x correctly, to obtain $\frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + c$ (penalise lack of constant of integration first time only)	M1, A1 (4)
(b)	Hence method : Uses $\cos 2x = 2\cos^2 x - 1$ to connect integrals	B1
	Obtains $\int x\cos^2 x\,dx = \frac{1}{2}\{\frac{x^2}{2} + \text{answer to part}(a)\} = \frac{x^2}{4} + \frac{x}{4}\sin 2x + \frac{1}{8}\cos 2x + k$	M1 A1 (3)
	Otherwise method	
	$\int x\cos^2 x\,dx = x(\frac{1}{4}\sin 2x + \frac{x}{2}) - \int \frac{1}{4}\sin 2x + \frac{x}{2}\,dx$ B1 for $(\frac{1}{4}\sin 2x + \frac{x}{2})$	B1, M1
	$= \frac{x^2}{4} + \frac{x}{4}\sin 2x + \frac{1}{8}\cos 2x + k$	A1 (3)
4 (a)	$r = 3$ (both circles)	B1
	Centres are at (2, 0) and (5, 0)	B1, B1 (3)
(b)	 1st circle correct quadrants centre on x axis	B1
	2nd circle correct quadrants centre on x axis	B1
	circles same size and passing through centres of other circle	B1 (3)
(c)	Finds circles meet at x = 3.5, by mid point of centres or by solving algebraically	M1
	Establishes $y = \pm\frac{3\sqrt{3}}{2}$, and thus distance is $3\sqrt{3}$.	M1, A1 (3)
	Or uses trig or Pythagoras with lengths 3, angles 60 degrees, or 120 degrees.	M1
	Complete and accurate method to find required distance	M1
	Establishes distance is $3\sqrt{3}$.	A1 (3)

Question Number	Scheme	Marks
5. (a)	Substitutes t = 4 to give V, = 1975.31 or 1975.30 or 1975 or 1980 (3 s.f)	M1 , A1 (2)
(b)	$\frac{dV}{dt} = -\ln 1.5 \times V$;= -800.92 or –800.9 or –801 M1 needs ln 1.5 term	M1 A1; A1 (3)
(c)	rate of decrease in value on 1st January 2005	B1 (1)
6, (a)	$\overrightarrow{AB} = \begin{pmatrix} c \\ d-5 \\ 10 \end{pmatrix} = k\begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}$ or $11 + 5\lambda = 21, \Rightarrow \lambda = 2$, $\therefore c = 4$, $d = 7$	M1 , A1 A1 (3)
(b)	$\begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix} \bullet \begin{pmatrix} 2\lambda \\ 5+\lambda \\ 11+5\lambda \end{pmatrix} = 0$	M1 A1
	$\therefore 4\lambda + 5 + \lambda + 55 + 25\lambda = 0$ $\therefore \lambda = -2$	M1 A1
	Substitutes to give the point P, $-4\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ (Accept (-4, 3, 1)	M1, A1 (6)
(c)	Finds the length of OA, or OB or OP or AB as $\sqrt{146}$ or $\sqrt{506}$ or $\sqrt{26}$ or $\sqrt{120}$ resp.	M1
	Uses area formula- either Area = $\frac{1}{2}\|\mathbf{AB}\| \times \|\mathbf{OP}\|$ or $= \frac{1}{2}\|\mathbf{OA}\| \times \|\mathbf{OB}\| \sin \angle AOB$ or = $\frac{1}{2}\|\mathbf{OA}\| \times \|A\mathbf{B}\| \sin \angle OAB$ or $= \frac{1}{2}\|AB\| \times \|\mathbf{OB}\| \sin \angle ABO$	M1
	= $\frac{1}{2}\sqrt{120}\sqrt{26}$ or $\frac{1}{2}\sqrt{146}\sqrt{506}\sin 11.86$ or $\frac{1}{2}\sqrt{146}\sqrt{120}\sin 155.04$ or $\frac{1}{2}\sqrt{120}\sqrt{506}\sin 13.10$	M1
	= 27.9	A1 (4)

Question Number	Scheme	Marks
7 (a)	As $V = \frac{4}{3}\pi r^3$, then $\dfrac{\mathrm{d}V}{\mathrm{d}r} = 4\pi r^2$	M1
	Using chain rule $\dfrac{dr}{dt} = \dfrac{dV}{dt} \div \dfrac{dV}{dr}$; $= \dfrac{k}{\frac{4}{3}\pi r^3} \times \dfrac{1}{4\pi r^2}$	M1 A1
	$= \dfrac{B}{r^5}$ *	A1 (4)
(b)	$\int r^5 dr = \int B dt$	B1
	$\therefore \dfrac{r^6}{6} = Bt + c$ (allow mark at this stage, does not need r =)	M1 A1 (3)
(c)	Use r =5 at t =0 to give c= $\dfrac{5^6}{6}$ or 2604 or 2600	M1
	Use r = 6 at t = 2 to give B = $\dfrac{6^5}{2} - \dfrac{5^6}{12}$ or 2586 or 2588 or 2590	M1
	Put t = 4 to obtain r^6 (approx 78000)	M1 A1
	Then take sixth root to obtain r = 6.53 (cm)	A1 (5)

Question Number	Scheme	Marks
8. (a)	$\frac{dx}{dt} = -\frac{1}{(1+t)^2}$ and $\frac{dy}{dt} = \frac{1}{(1-t)^2}$	B1, B1
	$\therefore \frac{dy}{dx} = \frac{-(1+t)^2}{(1-t)^2}$ and at $t = ½$, gradient is –9 M1 requires their dy/dt / their dx/dt and substitution of t.	M1 A1cao
	At the point of contact $x = \frac{2}{3}$ and $y = 2$	B1
	Equation is $y - 2 = -9\,(x - \frac{2}{3})$	M1 A1 (7)
(b)	**Either** obtain t in terms of x and y i,e, $t = \frac{1}{x} - 1$ *or* $t = 1 - \frac{1}{y}$ (or both)	M1
	Then substitute into other expression y = f(x) or x = g(y) and rearrange (or put $\frac{1}{x} - 1 = 1 - \frac{1}{y}$ and rearrange)	M1
	To obtain $y = \frac{x}{2x-1}$ *	A1 (3)
	Or Substitute into $\frac{x}{2x-1} = \frac{\frac{1}{(1+t)}}{\frac{2}{1+t} - 1}$	M1
	$= \frac{1}{2-(1+t)} = \frac{1}{1-t}$	A1
	$= y$ *	M1 (3)
(c)	Area $= \int_{\frac{2}{3}}^{1} \frac{x}{2x-1}\,dx$	B1
	$= \int \frac{u+1}{2u}\frac{du}{2} = \frac{1}{4}\int 1 + \frac{1}{u}\,du$ putting into a form to integrate	M1
	$= \left[\frac{1}{4}u + \frac{1}{4}\ln u\right]_{\frac{1}{3}}^{1}$	M1 A1
	$= \frac{1}{4} - (\frac{1}{12} + \frac{1}{4}\ln\frac{1}{3})$	M1
	$= \frac{1}{6} + \frac{1}{4}\ln 3$ or any correct equivalent.	A1 (6)

Question Number	Scheme	Marks
8. (c)	Or Area $= \int_{\frac{2}{3}}^{1} \frac{x}{2x-1} dx$	B1
	$= \int \frac{1}{2} + \frac{\frac{1}{2}}{2x-1} dx$ putting into a form to integrate	M1
	$= \left[\frac{1}{2}x + \frac{1}{4}\ln(2x-1)\right]_{\frac{2}{3}}^{1}$	M1A1
	$= \frac{1}{2} - \frac{1}{3} - \frac{1}{4}\ln\frac{1}{3} = \frac{1}{6} - \frac{1}{4}\ln\frac{1}{3}$	dM1 A1
		(6)
	Or Area $= \int \frac{1}{1-t}\frac{-1}{(1+t)^2} dt$	B1
	$= \int \frac{A}{(1-t)} + \frac{B}{(1+t)} + \frac{C}{(1+t)^2} dt$ putting into a form to integrate	M1
	$= \left[\frac{1}{4}\ln(1-t) - \frac{1}{4}\ln(1+t) + \frac{1}{2}(1+t)^{-1}\right]$	M1 A1ft
	= Using limits 0 and ½ and subtracting (either way round)	dM1
	$= \frac{1}{6} + \frac{1}{4}\ln 3$ or any correct equivalent.	A1
		(6)
	Or Area $= \int_{\frac{2}{3}}^{1} \frac{x}{2x-1} dx$ then use parts	B1
	$= \frac{1}{2}x\ln(2x-1) - \int_{\frac{2}{3}}^{1} \frac{1}{2}\ln(2x-1) dx$	M1
	$= \frac{1}{2}x\ln(2x-1) - [\frac{1}{4}(2x-1)\ln(2x-1) - \frac{1}{2}x]$	M1A1
	$= \frac{1}{2} - (\frac{1}{3}\ln\frac{1}{3} - \frac{1}{12}\ln\frac{1}{3} + \frac{1}{3})$	DM1
	$= \frac{1}{6} - \frac{1}{4}\ln\frac{1}{3}$	A1